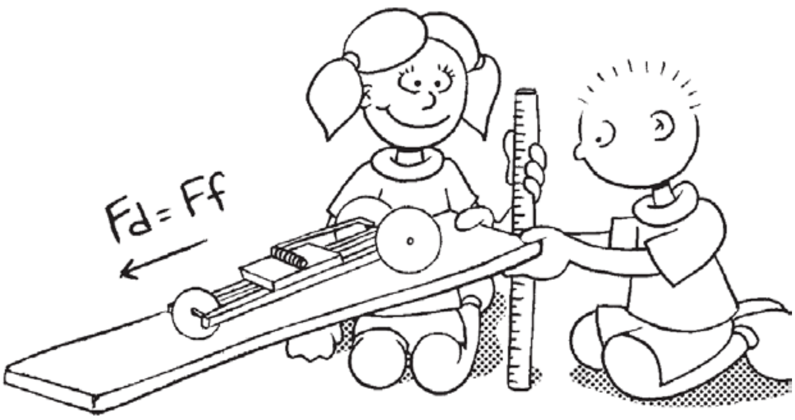
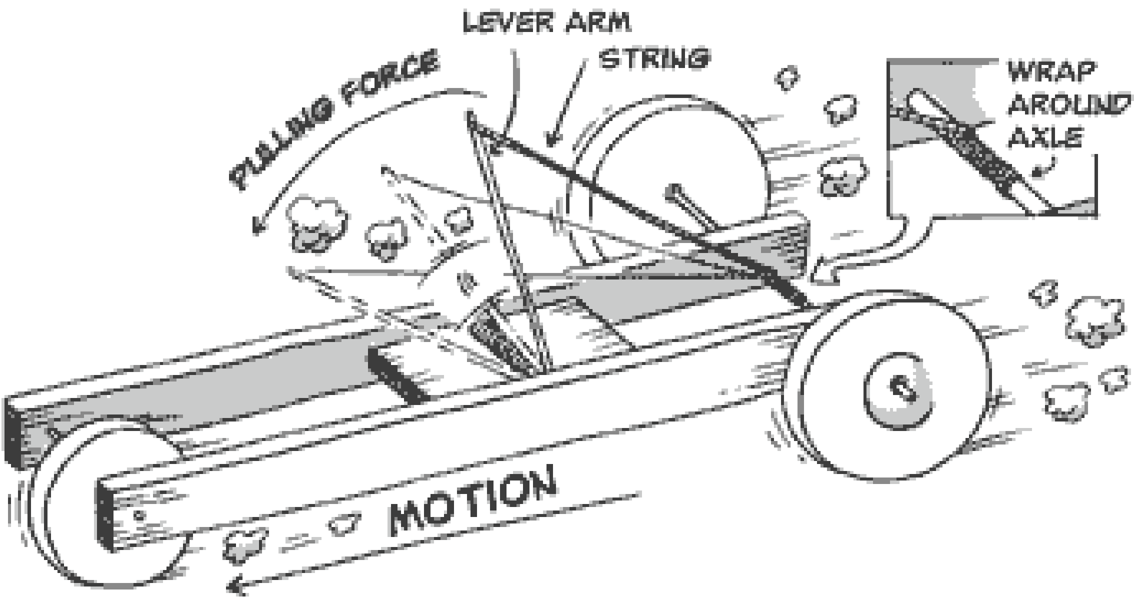
Mouse Trap Racer Calculations

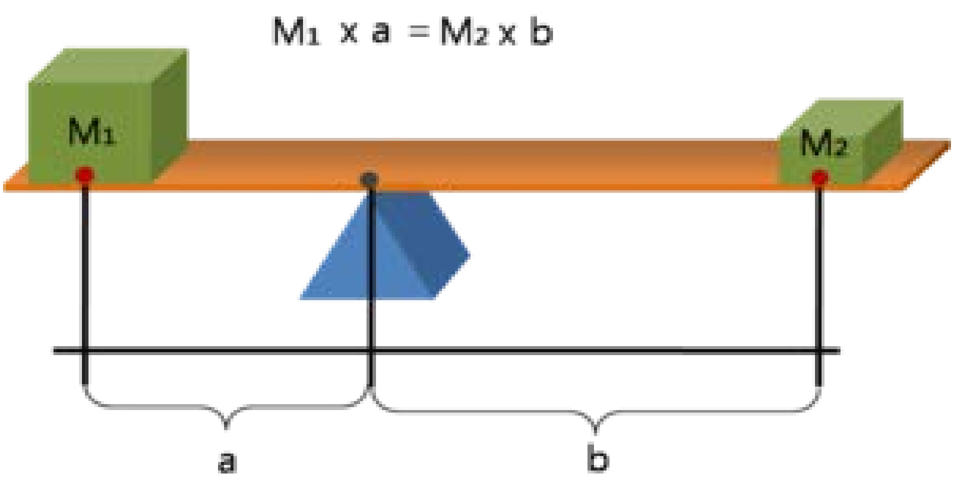
- Mechanical Advantage
- Rolling (Kinetic) Friction
- Rolling Friction Force
- Optimal Mechanical Advantage
- Vehicle Energy Efficiency





Mechanical Advantage Calculations

What is Mechanical Advantage:



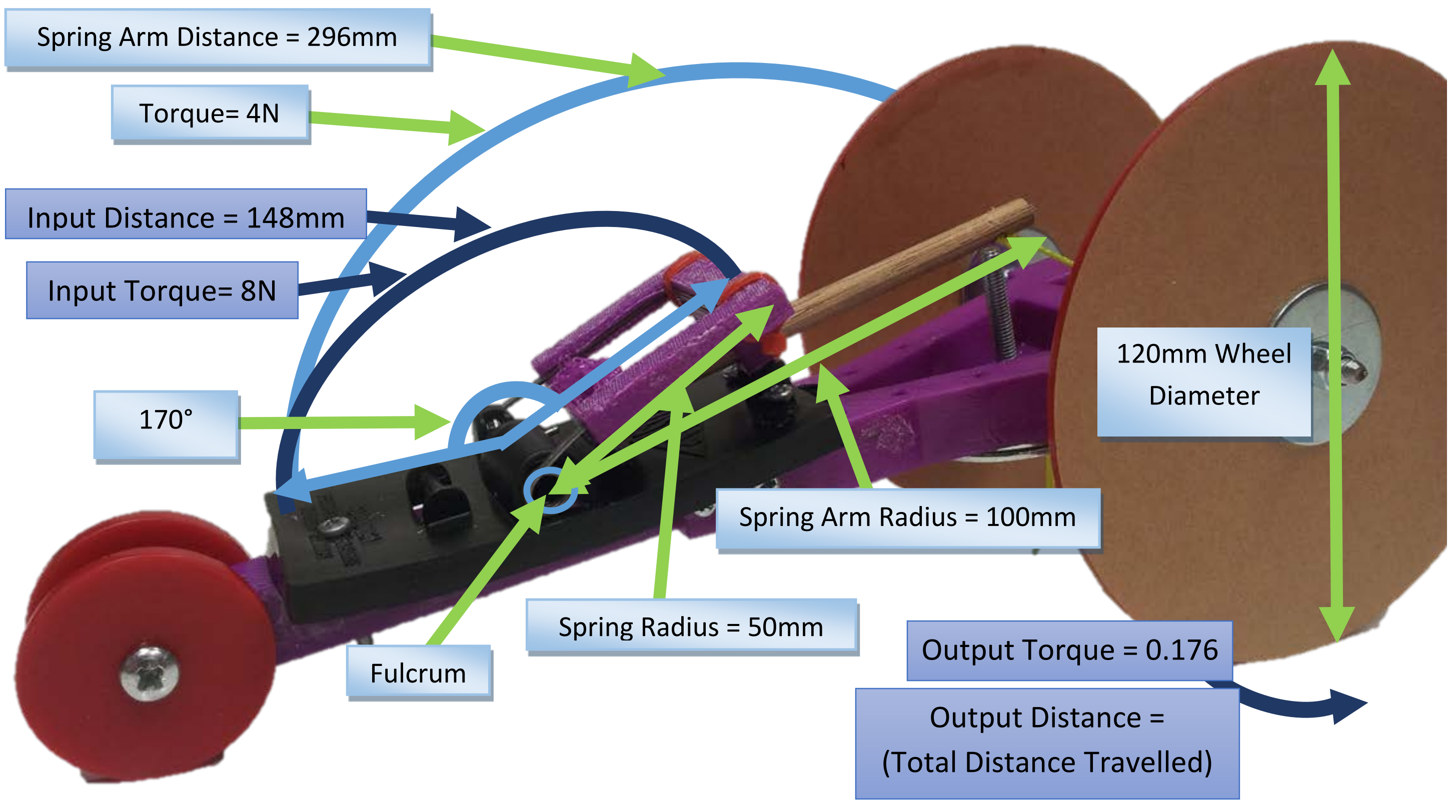
Machines allow you to do work more easiliy by spreading your effort over a greater distance/time.

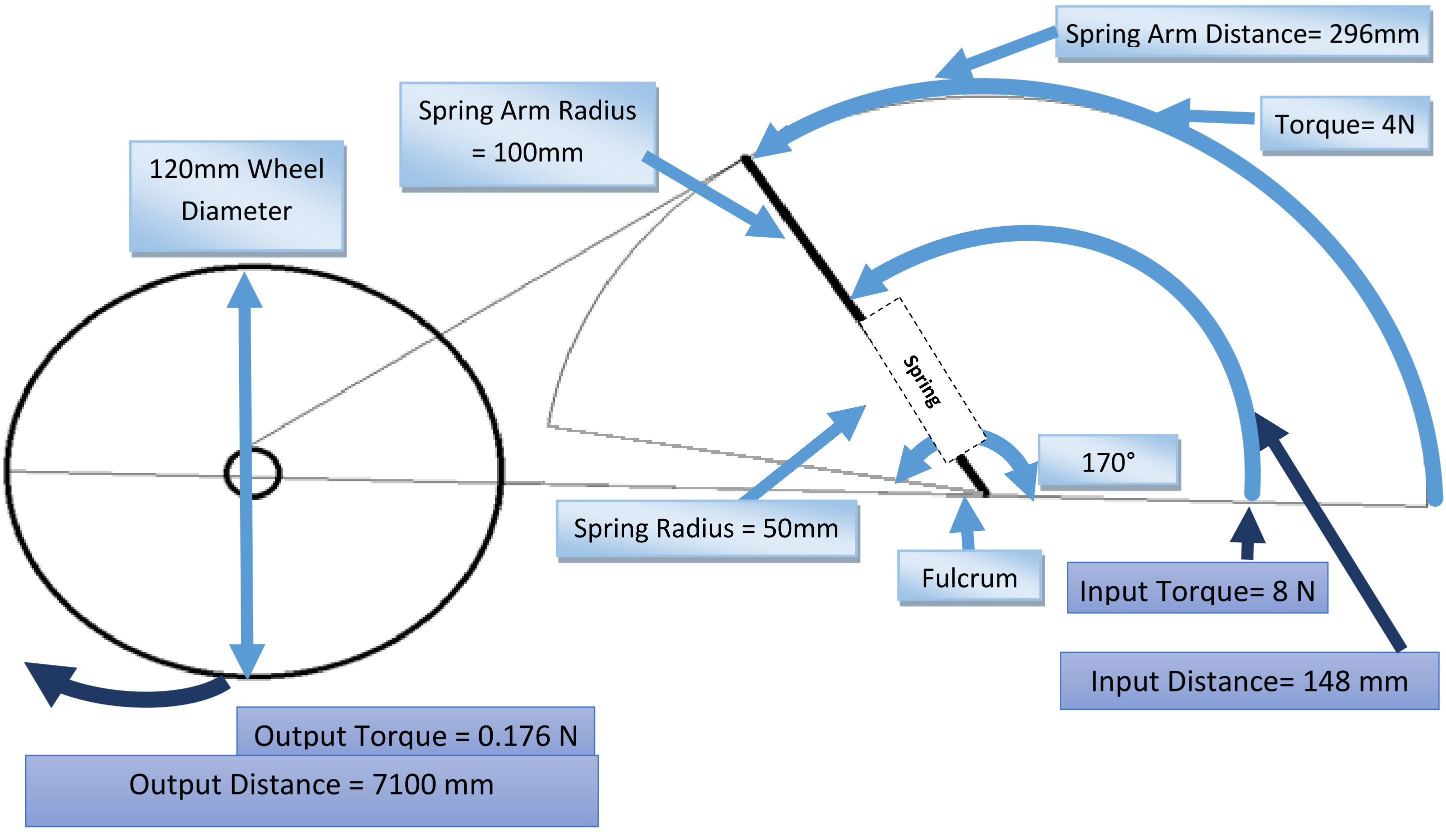
The mechanical advantage is the ratio of the input distance to the output distance or the ratio of the output force to the input force.

Archimede's law of levers is illustrated in the diagram above where a small Force (M_2) travelling a greater distance can "balance" a larger force (M_1) travelling a smaller distance.

A machine with a mechanical advantage greater than 1 means that it allows you to produce an output force greater than your applied input force- but you have to apply that smaller force over a greater distance- this is a biat like "first gear" in a car i.e. you amplify the force but reduce the distance.

A machine with a mechanical advantage less than 1 means that you must apply a greater input force over a smaller distance compared to the outputs produced- this is a bit like "fourth gear" in a car i.e. less torque or force output but greater distance.





$$Mechanical\ Advantage\ (MA) = \frac{Input\ Distance}{Output\ Distance}$$

$$MA = \frac{Travel\ Distance\ of\ Spring\ (mm)}{\left(\frac{Travel\ Distance\ of\ Spring\ Arm}{Circumference\ of\ Axle\ (mm)}\right) X\ Circumference\ of\ wheel\ (mm)}$$

Short Method: After rearranging above equation

$$MA_{TOTAL} = \frac{600}{Diameter\ Spring\ Arm\ X\ Diameter\ of\ Wheel}$$

$$MA_{TOTAL} = \frac{600}{200mm\ X\ 120mm}$$

$$MA_{TOTAL} = 0.025$$

Mechanical Advantage Calculations: Long Method

$$Axle\ Circumference = 2.\pi.r = 2\ X\ \ 3.14\ \ X\ 2.5mm = 15.7mm$$

$$Wheel\ Circumference = 2.\pi.r = 2\ X\ \ 3.14\ \ X\ 60mm = 376.8mm$$

1. Input Distance $= \frac{170°}{360°} \times 2\ \times 60mm \times 3.14$

$$= 178mm$$

2. Length of String Wrapped Around Axle $=$

$$= \ Total\ Spring\ Arm\ length\ \times \frac{170°}{360°} \times 2 \times \pi$$

$$= 100mm\ \times \frac{170°}{360°} \times 2\ \times 3.14$$

$$= 297mm$$

3. Number of Axle Rotations $= \frac{Length\ of\ String}{Axle\ Circumference}$

$$= \frac{297mm}{2\,\pi\ \times 2.5mm}$$

$$= 18.85\ revolutions$$

4. Output Distance $=$ Total Wheel Travel Distance

$$= \ No.\,Axle\ Rotations \times Wheel\ Circumference$$

$$= 18.85\ Revs\ \times\ \pi\ D$$

$$= 18.85\ \times 3.14 \times 120$$

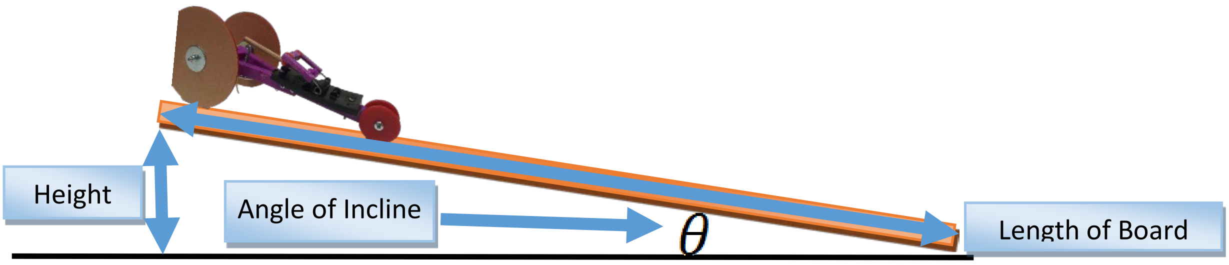
$$= 7102.68mm$$

$$Mechanical\ Advantage\ (MA) = \frac{Input\ Distance}{Output\ Distance}$$

$$= \frac{178mm}{7102mm}$$

$$= 0.025$$

Rolling Friction Inclined Plane Method



1. *Calculate Angle of Incline by using* $\sin(\theta) = \frac{Opposite}{Hyportenuse}$

$$\sin(\theta) = \frac{Opposite}{Hyportenuse} = \frac{Raised\ height}{Length\ of\ Board}$$

$$\sin(angle\ of\ incline) = \frac{16mm}{700mm}$$

$$= \sin^{-1}\frac{16}{700}$$

$$= 1.3$$

2. *Calculate Coefficient of Rolling Kinetic Friction* (μ_k)

Coefficient Rolling Friction (μ_k)

$$= Tan\left(angle\ of\ Incline\ (from\ 1.)\right)$$

$$\mu_k = \tan(1.3)$$

$$= 0.023$$

This means that about 2.3% of vertical force (weight) is required to overcome rolling friction in the horizontal plane.

Rolling Friction Force

The force required to overcome the rolling (kinetic) frictional opposing forces from the wheel/floor etc can be calculated from the mass of the vehicle and the coefficient of rolling friction as follows.

$$F_{Kinetic} = \mu_{Kinetic}.F_{Normal}$$

$$= 0.023\ X\ 0.270Kg\ X\ 9.8ms^{-2}$$

$$= 0.061\ Newtons$$

Optimal Mechanical Advantage

The design principle for a spring powered vehicle regarding Mechanical Advantage is that the torque at the drive wheels should be just enough to overcome rolling friction so that the spring's energy is dissipated as slowly as possible.

We can measure the average force of the spring with a simple spring balance (Force = Mass x 9.8m/s/s) or a Vernier Force Meter- this was found to be 8.0 Newtons on average.

The Mechanical Advantage required to reduce the force available at the spring arm (8.0 Newtons) to the Force required to overcome Rolling Friction can be calculated as follows:

$$MA\ (Theoretical\ Optimum) = \frac{Force_{Output}}{Force_{Input}}$$

$$= \frac{0.061\ Newtons}{8.0\ Newtons}$$

$$= 0.0076$$

Example Vehicle Specification:

If the vehicle wheel diameter is 120 what spring arm length is required to achieve the optimal MA of 0.0076? We can use the simplified MA formula:

$$MA_{TOTAL} = \frac{600}{Diameter\ Spring\ Arm\ X\ Diameter\ of\ Wheel}$$

$$0.0076 = \frac{600}{Diameter\ Spring\ Arm\ X\ 120\ mm}$$

$$Diameter\ Spring\ Arm = 658\ mm$$

Therefore the length or radius of the Spring Arm should be 658/2 = 329 mm

Vehicle Energy Efficiency

The energy efficiency of the vehicle is defined as the ratio of the desired output energy relative to the total input energy. In other words, the energy efficiency is the energy associated with the desired forward displacement relative to the stored potential energy in the wound up spring.

STEP 1: Calculate Work In: Work in is the applied force of the spring as it rotates. The torque applied by the spring was directly measured with a spring balance (Force = Mass x 9.8m/s/s) and found to be 8.0 Newtons on average. The length of the metal spring that transfers the force through the string to the axle was 50mm (0.05m) which is the radius of the arc of travel. The arm travels through 170° which equates to 1.2 Joules work.

$$Moment\ Arm\ Travel\ Distance = \frac{Travel\ Arc\,^{\circ}}{360^{\circ}} X 2\pi r$$

= 170° / 360° X 2 X 3.14 X 0.05m = 0.148m (148mm or 14.8cm)

$$Work\ (Nm\ or\ Joules) = Force\ (Newtons)\ X\ Distance\ (m)$$

= 8.0 N x 0.148 m = 1.2 Joules

STEP 2: Calculate Work Out: The energy required to move the vehicle forward against frictional forces is the work out.

$$F_{Kinetic} = \mu_{Kinetic}.F_{Normal} = 0.023\ X\ 0.270Kg\ X\ 9.8ms^{-2} = 0.061\ Newtons$$

$$Work\ (Nm\ or\ Joules) = Force\ (Newtons)\ X\ Distance\ (m)$$

= 0.061 Newtons x 6.48m (average displacement) = 0.40 Joules

STEP 3: Calculate Energy Efficiency:

$$Energy\ Efficiency\ (\eta) = \frac{Work\ out}{Work\ in}$$

$$Energy\ Efficiency\ (\eta) = \frac{0.40\ Joules}{1.2\ Joules} = 0.33 \text{ OR } 33\% \text{ Efficient}$$

This means that 33% of the input work associated with the stored spring energy is transferred to kinetic energy (movement) while overall frictional losses account for the remaining 66% (note that the work required to overcome the initial Static Frictional losses have been ignored but this would mean that this figure is an underestimation of the actual energy efficiency. Additionally the energy required to accelerate the vehicle to travelling speed is also ignored which would also cause the above efficiency calculation to be an underestimate.